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THE INFLUENCE OF SIZE-DISTRIBUTION PARAMETERS ON THE EVAPORATION OF POLYDISPERSE DILUTE SPRAYS

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Abstract — An analytical study has been performed of both diffusion-controlled and radiation-controlled (opaque droplets) evaporation of single-component polydisperse sprays. The overall evaporation characteristics of sprays of different initial size distributions were found to be correlated best by their initial Sauter mean diameter. In the case of diffusion-controlled evaporation, the representation of a polydisperse spray by a monodisperse spray of droplet size equal to the initial Sauter mean diameter of the polydisperse spray is a good assumption during the early stages of evaporation. When 40% of the volume of droplets has evaporated, the error of the monodisperse model reaches 20%. On the other hand, in the case of radiation-controlled evaporation, the monodisperse spray model yields good results for most of the period of evaporation of the spray.

NOMENCLATURE

a,	coefficient of size distribution, defined in equation (3);
b,	coefficient of size distribution;
B_T,	transfer number;
C_p,	specific heat of gases at constant pressure;
F_n,	number fraction of unevaporated droplets [$= n(t)/n(0)$];
F_v,	volume fraction of unevaporated droplets [$= V(t)/V(0)$];
$f(x,t)$,	number of droplets of diameter x at time t;
h_{fg},	heat of vaporization;
k,	exponent of droplet diameter in the evaporation law;
K,	thermal conductivity;
n,	total number of droplets at time t;
n_0,	total number of droplets at time $t = 0$;
Q_r,	heat of reaction;
t,	time;
T,	temperature;
V,	volume of unevaporated droplets;
x,	diameter of droplet;
$\bar{x}_{10}$,	linear mean diameter of spray;
$\bar{x}_{20}$,	surface mean diameter of spray;
$\bar{x}_{30}$,	volume mean diameter of spray;
$\bar{x}_{32}$,	Sauter mean diameter of spray;
y_{10},	$x/\bar{x}_{10}$;
y_{32},	$x/\bar{x}_{32}$;
Y_o,	mass fraction of oxidizer;
W_o,	molecular weight of oxidizer;
z,	defined in equation (33);
γ,	constant of size distribution, defined in equation (1);
Γ,	gamma function;
ε,	constant of size distribution, defined in equation (1);
δ,	defined in equation (18);
λ,	coefficient of evaporation;
ρ_L,	density of liquid droplet;
σ,	defined in equation (18);
σ_{ij},	$= \bar{x}_{ij}(t)/\bar{x}_{ij}(0)$;
τ_{1j},	dimensionless evaporation time based on $\bar{x}_{ij}$ [see equation (34)].

Subscripts

∞,	conditions far away from droplet;
L,	conditions on the surface of droplet.

INTRODUCTION

THE EVAPORATION of fuel sprays in diesel and other reciprocating engines strongly influences the ignition and subsequent combustion of the fuel in these engines. Modelling of the evaporation and combustion of polydisperse fuel sprays requires considerable numerical effort. This problem may be simplified, however, if the polydisperse fuel spray can be represented by an "equivalent" monodisperse spray. The validity of such representation requires answers to the following questions: (1) What is the droplet size of the theoretical monodisperse spray that best represents the evaporation of the corresponding polydisperse spray, and (2) what is the error in the predicted amount of fuel evaporated if the monodisperse spray assumption is evoked?

This investigation was directed towards providing answers to these questions by considering the evaporation of polydisperse sprays of different size distributions. Some of the empirically determined size distributions of sprays used were: the Rosin–Rammler distribution [1], the Nukiyama–Tanasawa distribution [2], and the Hiroyasu–Kadota distribution [3, 4]. The Tanasawa–Toyoda distribution [5] (a limiting form of the Nukiyama–Tanasawa distribution) and the Hiroyasu–Kadota distribution are considered applicable to diesel engine fuel-injection systems.

The diffusion-controlled evaporation of polydisperse sprays has been considered by Probert [6], Tanasawa and Tesima [7], and Williams [8, 9]. Probert [6] examined sprays with size distributions given by the Rosin–Rammler distribution, whereas Tanasawa and Tesima [7] examined a variety of size distributions that are generally described by equation (1) (see subsequent section). In both studies the volume of unevaporated droplets of the spray was determined as a function of time by graphical integration of the resultant equations. Williams [8, 9] developed an elegant theory of the evaporation of a polydisperse spray in a variable-area quasi-one-dim. rocket chamber. The combustion efficiency was determined as a function of chamber length for a spray conforming to the Rosin–Rammler size distribution. It must be noted that presently the Rosin–Rammler distribution is rarely utilized. Ingebo [10, 11] found this distribution to represent poorly the droplet size distributions of sprays in turbulent air streams.

Dickinson and Marshall [12] considered the influence of the relative velocity of the droplets on the rate of evaporation of polydisperse sprays of various size distributions which included log normal and square-root normal distributions. For the case of negligible relative velocity of the droplets, i.e. diffusion-controlled evaporation, they concluded that no mean diameter can adequately characterize the evaporation characteristics of polydisperse sprays.

The present study may be considered as an extension of the Tanasawa and Tesima [7] study. In the present study, both diffusion- and radiation-controlled evaporation has been considered. Radiation in diesel engines is very significant [13, 14] and is expected to influence the evaporation rate of droplets strongly after combustion has been initiated. The influence of radiation increases with an increase in the size of the droplet, as well as with an increase in the temperature of its surroundings. For an opaque droplet in a low-convective flow environment, the ratio of radiative to convective heat transfer is directly proportional to the droplet diameter. In the case of a 100 μm n-decane fuel droplet in a 2500 K combustion gas environment this ratio is about 0.6. However, in spray combustors, which are high-convective flow systems, radiation is not expected to control the evaporation of droplets.

The problem considered here is the steady-state evaporation of polydisperse sprays of different size distributions in a constant volume chamber under steady environmental conditions. All the droplets in a spray are assumed to start evaporating at time $t = 0$. The present treatment of the evaporation of sprays still remains idealized because the effects of (a) droplet interactions, (b) relative velocity between the liquid and gas phases, and (c) droplet preheating are not considered.

DROPLET SIZE DISTRIBUTION IN SPRAYS

The size distribution of droplets in sprays produced by diesel engine, gas turbine and other injection systems may be correlated by the general expression [8, 9, 15]:

$$\frac{\mathrm{d}n}{n} = a x^{\varepsilon} \exp[b x^{\gamma}]\,\mathrm{d}x, \tag{1}$$

where n is the total number of droplets and $\mathrm{d}n/n$ represents the fraction of the number of droplets of a diameter between x and $x + \mathrm{d}x$. Constants ε and γ are experimentally determined parameters that depend on the type of injector used. The coefficient a can be evaluated from the total number of droplets, i.e.

$$n \equiv \int_0^{\infty} \mathrm{d}n \tag{2}$$

Thus,

$$a = \gamma (b)^{(\varepsilon+1)/\gamma} \Big/ \Gamma\left(\frac{\varepsilon+1}{\gamma}\right) \tag{3}$$

where the Gamma function is defined as:

$$\Gamma(k) = \int_0^{\infty} x^{k-1} \exp(-x)\,\mathrm{d}x. \tag{4}$$

The coefficient b can be expressed in terms of a characteristic mean diameter of the polydisperse spray. In the field of spray evaporation and combustion, droplet mean diameters of primary importance are: the linear mean diameter $\bar{x}_{10}$, the surface mean diameter $\bar{x}_{20}$, the volume mean diameter $\bar{x}_{30}$, and the Sauter mean diameter $\bar{x}_{32}$. These diameters are defined by the general formula

$$\bar{x}_{ij} \equiv \left[\int_0^{\infty} x^i\,\mathrm{d}n \Big/ \int_0^{\infty} x^j\,\mathrm{d}n\right]^{1/(i-j)} \tag{5}$$

Arbitrarily choosing the Sauter mean diameter as the characteristic mean diameter of the spray droplets, the coefficient b is related to this mean diameter by the expression:

$$\bar{x}_{32} \equiv \int_0^{\infty} x^3\,\mathrm{d}n \Big/ \int_0^{\infty} x^2\,\mathrm{d}n = \left(\frac{1}{b}\right)^{1/\gamma} \Gamma\left(\frac{\varepsilon+4}{\gamma}\right) \Big/ \Gamma\left(\frac{\varepsilon+3}{\gamma}\right). \tag{6}$$

Using equations (3) and (6), the coefficients a and b of equation (1) are eliminated to give:

$$\frac{\mathrm{d}n}{n} = \frac{\gamma}{\Gamma\left(\frac{\varepsilon+1}{\gamma}\right)} \left[\frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)}\right]^{\varepsilon+1} (y_{32})^{\varepsilon} \times \exp\left[-\left(\frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)} y_{32}\right)^{\gamma}\right] \mathrm{d}y_{32}, \tag{7}$$

where $y_{32} = x/\bar{x}_{32}$. In equation (7) the diameter of the droplets was normalized by the Sauter mean diameter. Similar expressions may be obtained using one of the other mean diameters of the spray to normalize the droplet diameter.

The droplet volume distribution can be obtained from the droplet number distribution. Assuming spherical droplets, the volume of droplets of diameters between x and $x + \mathrm{d}x$ is

$$\mathrm{d}V = \frac{\pi}{6} x^3 \,\mathrm{d}n = \frac{\pi}{6}\, a n x^{\varepsilon+3} \exp[-bx^{\gamma}]\,\mathrm{d}x. \quad (8)$$

The total volume of the droplets in the spray is

$$V \equiv \int_0^{\infty} \mathrm{d}v = \frac{\pi}{6} an \left(\frac{1}{b}\right)^{(\varepsilon+4)/\gamma} \Gamma\left(\frac{\varepsilon+4}{\gamma}\right). \quad (9)$$

Using equations (6) and (9) in (8), the volume fraction of droplets of a certain size range may be expressed in terms of y_{32} as

$$\frac{\mathrm{d}V}{V} = \frac{\gamma}{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)} \left[\frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)}\right]^{\varepsilon+4} y_{32}^{\varepsilon+3}$$

$$\times \exp\left[-\left(\frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)} y_{32}\right)^{\gamma}\right] \mathrm{d}y_{32}. \quad (10)$$

It is apparent from the above equations that for a given injector (i.e. given values of ε and γ) the number and volume distributions of droplets is determined by either the Sauter mean diameter or any other mean diameter of the spray.

The number distributions and volume distributions for various values of ε and γ are shown in Figs. 1 and 2 respectively.

A MODEL OF SPRAY EVAPORATION

Governing equations

Let $f(x, t)$ be the number of droplets of diameter x at time t. Assuming no interaction between particles, such as coagulation of particles, and that evaporation is the only process that the droplets undergo, then a balance of the number of particles of diameter x at times t and $t + \mathrm{d}t$ gives

$$\frac{\partial f}{\partial t} = -\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\partial f}{\partial x}. \quad (11)$$

Equation (11) is a limited form of the general spray equation presented in [9]. To proceed with its solution, the variation of the diameter of the droplets with respect to time must be specified. In the present study, the vaporization process of individual droplets is described by:

$$[x(t)]^{k+1} = [x(0)]^{k+1} - \lambda t. \quad (12)$$

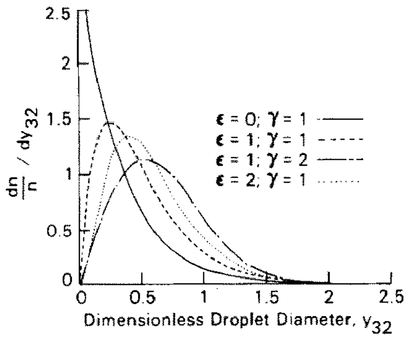

FIG. 1. Number distributions of spray droplets for various combinations of the spray constants ε and γ.

Consequently

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\lambda}{(k+1)x^k} \quad (13)$$

and

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{6}\pi x^3\right) = -\frac{\pi\lambda}{2(k+1)} x^{2-k}, \quad (14)$$

where λ is the coefficient of evaporation and $0 \le k \le 1$. For diffusion-controlled evaporation, $k = 1$ [16, 17]. This is known as the "d^2-law" because the square of the diameter of the droplet varies linearly with time [see equation (12)]. In this case the rate of change of the diameter of the droplet is inversely proportional to its diameter, whereas the rate of change of the volume of the droplet is directly proportional to its diameter. For radiation-controlled evaporation of opaque droplets, $k = 0$. In this case, the diameter of the droplet varies linearly with time; consequently the rate of change of the diameter of the droplet is constant. On the other hand, the rate of change of the volume of the droplet is proportional to the square of the diameter.

The assumptions underlying equation (12) are:

(1) The evaporation is quasi-steady.

(2) The droplet consists of a single-component fuel.

(3) The evaporation of one droplet is not influenced by other droplets.

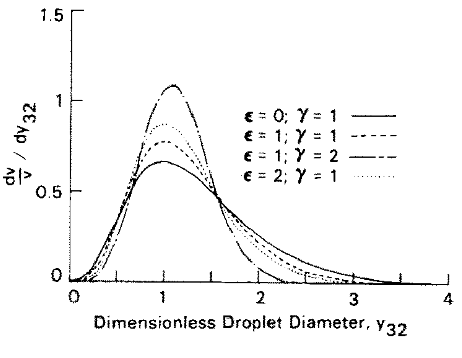

FIG. 2. Volume distributions of spray droplets for various combinations of the spray constants ε and γ.

(4) The environmental conditions (i.e. pressure, temperature and gaseous species concentrations) are steady and uniform.

(5) The convective processes are negligible.

For steady conditions the evaporation constant λ is constant. For example, for diffusion-controlled evaporation with constant properties, the coefficient of evaporation λ is given by [9]:

$$\lambda = \frac{8K}{C_p \rho_L} \ln(1 + B_T) \tag{15}$$

where the transfer number B_T is given by:

$$B_T = \begin{cases} \dfrac{C_p(T_\infty - T_L)}{h_{fg}}, & \text{for evaporation without combustion} \\ \dfrac{C_p(T_\infty - T_L) + Q_r Y_{o\infty}}{h_{fg}}, & \text{for evaporation with combustion.} \end{cases}$$

The effects of interactions among droplets on their combustion characteristics have been studied by numerous investigators [18–26]. Experimental studies on suspended droplets [18, 19, 20, 21] and arrays of freely falling droplets [22, 23] found that the "d^2-law" is still applicable; however the coefficient of evaporation is significantly reduced. These results were theoretically substantiated by Brzustowski *et al.* [24], Umemura *et al.* [25] and Labowsky [26]. The last author concluded that, while not rigorous, the "d^2-law" does approximate the burning history of an interacting droplet, especially during the initial period of combustion.

The conditions of individual burning of single droplets versus collective or group burning of the droplets were studied by Suzuki and Chiu [27], Labowsky and Rosner [28] and Chiu and Liu [29]. These theoretical studies indicated that individual droplet burning is encountered rarely in spray combustors. Qualitative experimental confirmations of these results were obtained by Chigier and McCreath [30], Twardus and Brzustowski [31] and Sangiovanni and Dodge [32]. The last study showed that group combustion is sustained even at droplet spacings of the order of 25 droplet diameters.

Substituting equation (13) in (11) yields

$$\frac{\partial f}{\partial t} - \frac{\lambda}{(k+1)x^k} \frac{\partial f}{\partial x} = 0. \tag{16}$$

The initial condition based on the general droplet size distribution of equation (1) is

$$f(x,0) = a n_o x^{\varepsilon} \exp[-b x^{\gamma}]. \tag{17}$$

Solution

Equation (16) subjected to the initial condition (17) may be solved by using the following independent variable transformations:

$$\delta = [x^{k+1} + (1 + \lambda)t]^{1/(k+1)}$$

and

$$\sigma = [x^{k+1} + \lambda t]^{1/(k+1)} \tag{18}$$

Using equation (18), equation (16) is transformed to

$$\frac{\mathrm{d}f}{\mathrm{d}\delta} = 0. \tag{19}$$

Hence the solution is

$$f(x,t) = \phi(\sigma) = \phi([x^{k+1} + \lambda t]^{1/(k+1)}). \tag{20}$$

The unknown function $\phi(\sigma)$ is determined from the initial condition, i.e. at

$$t = 0, \qquad \phi(\sigma) = f(x,0). \tag{21}$$

Thus the time dependence of the size distribution of an evaporating spray with an initial size distribution given by equation (1) is

$$f(x,t) = a n_0 [x^{k+1} + \lambda t]^{\varepsilon/(k+1)} \times \exp[-b(x^{k+1} + \lambda t)^{\gamma/(k+1)}]. \tag{22}$$

Using equation (22), the variation with time of several pertinent spray parameters, such as the number and volume of unevaporated droplets of the spray and the various mean diameters of the spray, can be evaluated as follows:

$$n(t) = \int_0^\infty f(x,t)\mathrm{d}x, \tag{23}$$

$$V(t) = \frac{\pi}{6} \int_0^\infty x^3 f(x,t)\mathrm{d}x, \tag{24}$$

$$\bar{x}_{ij}(t) = \left[\frac{\int_0^\infty x^i f(x,t)\mathrm{d}x}{\int_0^\infty x^j f(x,t)\mathrm{d}x} \right]^{1/(i-j)} \tag{25}$$

The generalized integral H(a, b, c, z)

Evaluation of the above spray parameters requires the evaluation of an integral of the form:

$$I = \int_0^\infty x^s [x^{k+1} + \lambda t]^{\varepsilon/(k+1)} \times \exp[-b(x^{k+1} + \lambda t)^{\gamma/(k+1)}]\mathrm{d}x \tag{26}$$

where s is an integer equal to 0, 1, 2 or 3.

Defining

$$u \equiv b(x^{k+1} + \lambda t)^{\gamma/(k+1)} - b(\lambda t)^{\gamma/(k+1)}$$

and

$$z \equiv b(\lambda t)^{\gamma/(k+1)} \tag{27}$$

equation (26) becomes

$$I = \left(\frac{1}{\gamma}\right)\left(\frac{1}{b}\right)^{(s+\varepsilon+1)/\gamma} H\left(\frac{k+1}{\gamma}, \frac{s-k}{k+1}, \frac{k+\varepsilon+1}{\gamma}, z\right), \tag{28}$$

where the function H is defined* as:

$$H(a,b,c,z) \equiv \exp(-z) \times \int_0^x [(u+z)^a - z^a]^b (u+z)^{c-1} \exp(-u)\mathrm{d}u. \quad (29)$$

The temporal variation of the linear mean diameter, surface mean diameter, volume mean diameter, Sauter mean diameter, and the number and volume of unevaporated droplets in the spray are then given by:

$$\bar{x}_{iJ}(z) = \bar{x}_{iJ}(0) \left[\frac{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right) H\left(\frac{k+1}{\gamma}, \frac{i-k}{k+1}, \frac{k+\varepsilon+1}{\gamma}, z\right)}{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right) H\left(\frac{k+1}{\gamma}, \frac{j-k}{k+1}, \frac{k+\varepsilon+1}{\gamma}, z\right)}\right]^{1(i-j)}, \quad (30)$$

$$n(z) = \frac{n(0)}{\Gamma\left(\frac{\varepsilon+1}{\gamma}\right)} H\left(\frac{k+1}{\gamma}, \frac{-k}{k+1}, \frac{k+\varepsilon+1}{\gamma}, z\right), \quad (31)$$

$$V(z) = \frac{V(0)}{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)} H\left(\frac{k+1}{\gamma}, \frac{3-k}{k+1}, \frac{k+\varepsilon+1}{\gamma}, z\right), \quad (32)$$

where z may be expressed as a function of the initial Sauter mean diameter of the spray, $\bar{x}_{32}(0)$, as:

$$z \equiv \frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)} \tau_{32}^{\gamma/(k+1)} \quad (33)$$

where

$$\tau_{32} = \frac{\lambda t}{[\bar{x}_{32}(0)]^{k+1}}. \quad (34)$$

Numerical evaluation of H(a, b, c, z)

For a given initial spray droplet distribution (i.e. specified values of ε and γ) and a given evaporation law (i.e. either $k = 0$ or $k = 1$), the integral $H(a, b, c, z)$ was evaluated for various values of z and consequently the dimensionless evaporation time τ_{32}. The integration was performed using Romberg's numerical method [33]. Because the upper limit of integration is infinite, the integral was evaluated for successively larger upper limits until the values of two successive integrations agreed within a tolerance of 0.01%.

RESULTS AND DISCUSSION

Typical temporal variations of the number distribution and volume distribution of droplets in an evaporating spray are shown in Figs. 3 and 4 respectively. The constants of the initial distribution are: $\varepsilon = 1.0$, $\gamma = 1.0$. Evaporation is assumed to obey the "d^2-law" (i.e. $k = 1.0$). Figure 3 shows that as the time of evaporation progresses, the maximum in the number distribution moves towards smaller droplet diameters. The size of droplets of maximum number-fraction, y_{32}^M, is given by

$$y_{32}^M = \frac{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)} \left[\left(\frac{\varepsilon}{\gamma}\right)^{(k+1)/\gamma} - \left(\frac{\Gamma\left(\frac{\varepsilon+4}{\gamma}\right)}{\Gamma\left(\frac{\varepsilon+3}{\gamma}\right)}\right)^{k+1} \tau_{32}\right]^{1/(k+1)}. \quad (35)$$

Thus, in the present case (i.e. $\varepsilon = 1.0$, $\gamma = 1.0$, $k = 1.0$) the mode occurs at $y_{32}^M = 0$ when $\tau_{32} \geq 0.0625$. The shift of the mode towards smaller diameters as evaporation progresses indicates that the diameters of the smaller droplets decrease faster than the diameters of the bigger droplets. This is expected, since the rate of change of the diameter of a droplet is inversely proportional to its diameter [see equation (13) for $k = 1$]. On the other hand, the maximum of the volume distribution moves towards larger droplet sizes as evaporation progresses. This results because the rate of change of the volume of a droplet evaporating according to the "d^2-law" is proportional to its diameter.

The temporal variations of the number and volume of unevaporated fuel droplets and of the various pertinent mean diameters of the spray droplets are

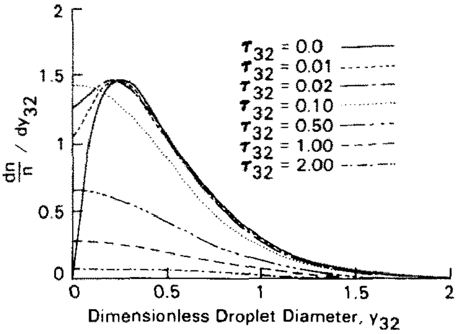

FIG. 3. Temporal variations of the number distribution of a spray ($\varepsilon = 1.0$, $\gamma = 1.0$) undergoing diffusion-controlled evaporation.

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*The definition of the integral $H(a, b, c, z)$ differs slightly from the definition given by Williams [9].

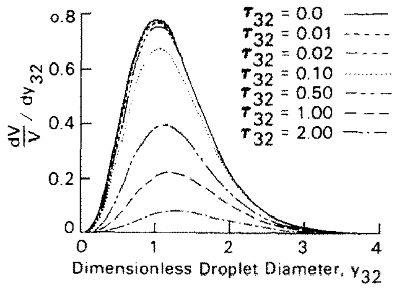

FIG. 4. Temporal variations of the volume distribution of a spray ($\varepsilon = 1.0$, $\gamma = 1.0$) undergoing diffusion-controlled evaporation.

shown in Fig. 5 for the case of diffusion-controlled evaporation of a spray of initial droplet size distribution defined by: $\varepsilon = 1.0, \gamma = 1.0$. These parameters, which characterize the evaporating polydisperse spray, are normalized by their initial values. As expected, Fig. 5 shows that the number and volume of unevaporated fuel droplets decrease with increasing evaporation time. On the other hand, the various mean diameters of the droplets, despite an initial small decrease in their values (with the exception of the Sauter mean diameter), increase with increasing evaporation time.

Figure 6 shows the temporal variations of the parameters corresponding to Fig. 5 for the case of diffusion-controlled evaporation of a spray having initial droplet size distribution defined by $\varepsilon = 0.0$, $\gamma = 1.0$. In this case the various mean diameters of the droplets increase monotonically with increasing evaporation time.

The general tendency of the mean diameters of the spray droplets to increase in value during most of the evaporation period is partly a consequence of the "d^2-law" of evaporation, which results in the smaller droplets decreasing in size at a faster rate than the bigger droplets. The temporal variation of the mean diameters of the droplets also depends on the initial size distribution. For example, in the limiting case of a monodisperse spray, the mean diameter of the droplets decreases monotonically with increasing evaporation time. In this regard, Dickinson and Marshall [12] concluded that for diffusion-controlled evaporation there is a general tendency for the average diameter of the remaining droplets to increase in the moderately or highly nonuniform sprays and to decrease in the more uniform sprays.

In the case of radiation-controlled evaporation of sprays ($k = 0$), the temporal variations of their pertinent parameters are shown in Fig. 7 for an initial size distribution defined by $\varepsilon = 1.0$, $\gamma = 1.0$.

The radiation-controlled evaporation law dictates that all droplets, independent of size, decrease in size at the same rate. Consequently it is expected that in general the mean diameters of the droplets decrease with increasing evaporation time. The initial distribution defined by $\varepsilon = 0.0$, $\gamma = 1.0$ is a special case. In this case, at any instant during the evaporation period the number fraction of unevaporated droplets is equal to their volume fraction and the mean diameters of the droplets are time-invariant.

In spray combustion, one of the most important parameters to study is the amount of fuel evaporated as a function of time. Often the evaporation process is the rate-determining step in the combustion of liquid fuels. In this case the rate of evaporation is equal to the rate of combustion.

In many processes involving polydisperse sprays, such as evaporation and combustion of sprays, it is more desirable to work with the mean diameters of the droplets than with the complete droplet size distribution. As was stated earlier, in the field of evaporation and combustion of sprays the important mean diameters of the spray droplets are the linear mean diameter, the surface mean diameter, the volume mean diameter, and the Sauter mean diameter. The question thus arises as to which of these mean diameters, if any, may adequately characterize the evaporation of the spray.

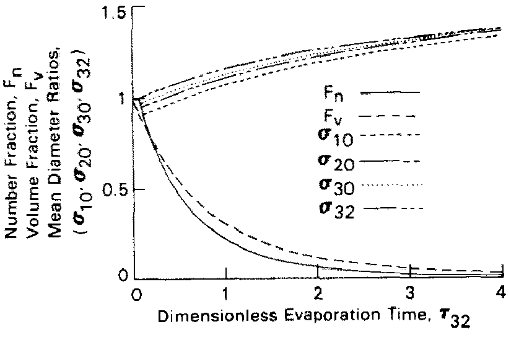

FIG. 5. Temporal variations of the number and volume of unevaporated droplets and of the various mean diameters of a spray undergoing diffusion-controlled evaporation. (Initial size distribution: $\varepsilon = 1.0, \gamma = 1.0$.)

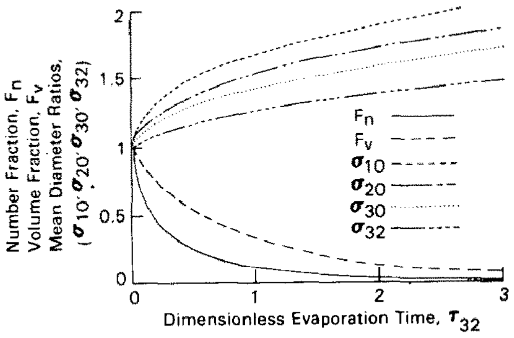

FIG. 6. Temporal variations of the number and volume of unevaporated droplets and of the various mean diameters of a spray undergoing diffusion-controlled evaporation. (Initial size distribution: $\varepsilon = 0.0, \gamma = 1.0$.)

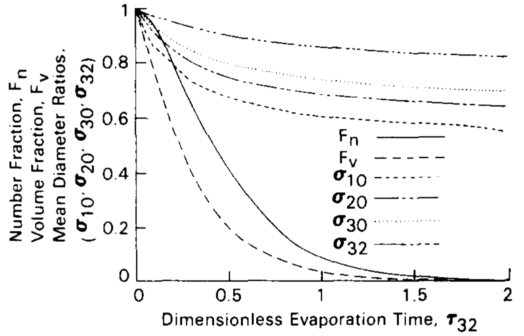

FIG. 7. Temporal variations of the number and volume of unevaporated droplets and of the various mean diameters of a spray undergoing radiation-controlled evaporation. (Initial size distribution: $\varepsilon = 1.0, \gamma = 1.0$.)

To resolve this question, the evaporation rates of four sprays of different initial size distribution were compared for four cases. In each case the initial value of one of the pertinent mean diameters of the spray droplets was kept the same. For example, in the first case the evaporation of the sprays was compared keeping their initial-droplet linear mean diameters the same, whereas in the second case the initial surface mean diameters were kept the same.

These comparisons at constant mean diameters can be accomplished by considering the volume fraction F_v of unevaporated droplets of each of four sprays as a function of dimensionless time, which is based on the initial mean diameter. Thus, a plot of F_v vs τ_{10} for the four sprays considered shows a comparison of the evaporation of the sprays at constant linear mean diameter $\bar{x}_{10}$. Figure 8 shows comparisons of the evaporation of sprays of various size distributions for the same initial-droplet linear mean diameter, surface mean diameter, volume mean diameter and Sauter mean diameter respectively. These are plots of the volume fraction F_v of unevaporated fuel droplets as a function of the corresponding normalized evaporation time for various combinations of the size distribution constants ε and γ. In all cases shown, evaporation obeys the "d^2-law", i.e. diffusion-controlled evaporation prevails.

To aid with the subsequent discussion, Table 1 shows the ratios of each of the initial mean diameters to the initial Sauter mean diameter for the combinations of values of ε and γ considered in Fig. 8. Also

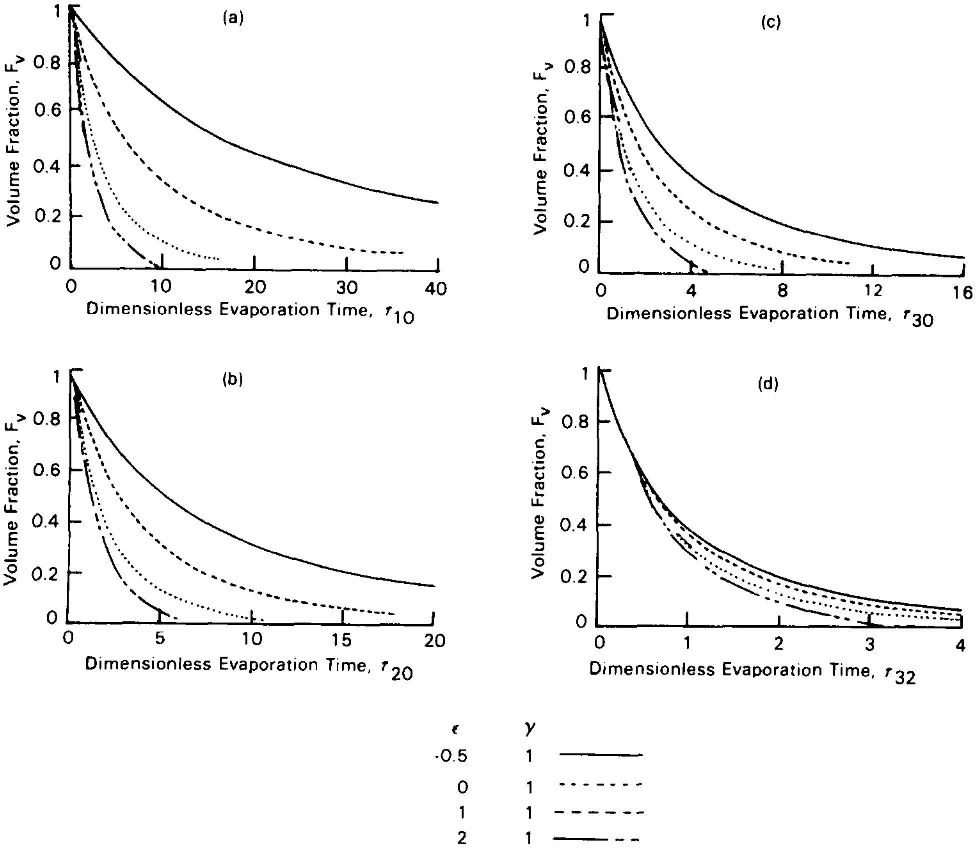

FIG. 8. Comparisons of the diffusion-controlled evaporation of sprays of various initial size distributions. (a) Same initial linear diameters; (b) same initial surface mean diameters; (c) same initial volume mean diameters; (d) same initial Sauter mean diameters.

Table 1

ε	γ	$\frac{\bar{x}_{10}}{\bar{x}_{32}}$	$\frac{\bar{x}_{20}}{\bar{x}_{32}}$	$\frac{\bar{x}_{30}}{\bar{x}_{32}}$
−0.5	1.0	0.200	0.346	0.493
0.0	1.0	0.333	0.471	0.606
1.0	1.0	0.500	0.612	0.721
2.0	1.0	0.600	0.693	0.783
Data [34]		0.21	0.31	0.46

tabulated are the corresponding ratios compiled from a large number of experimental measurements on various sprays at various environmental conditions [34].

Comparison of Fig. 8 shows that the best correlation of the evaporation of the four different sprays is obtained by keeping their initial-droplet Sauter mean diameters the same [Fig. 8(d)]. In this case, the calculated volume fraction of unevaporated fuel droplets at any given time of evaporation is not appreciably influenced by the initial distribution within the spray. This is especially true during the early stages of evaporation, where the volume fraction of unevaporated droplets may be considered nearly independent of initial-droplet size distribution within the spray. The influence of the initial distribution on the amount evaporated is strongest during the final stage of evaporation.

On the other hand, it is apparent from Figs. 8(a), (b) and (c) that the evaporation rates of the various sprays of the same initial-droplet linear mean diameter, surface mean diameter or volume mean diameter vary considerably for different values of the size distribution constants ε and γ. These combinations of the size distribution constants result in different relations between the corresponding initial mean diameters ($\bar{x}_{10}$, $\bar{x}_{20}$, $\bar{x}_{30}$) and the initial Sauter mean diameter $\bar{x}_{32}$, as shown in Table 1. It is apparent, furthermore, that the lower the initial Sauter mean diameter of the droplets, the faster its evaporation rate.

It may therefore be concluded that the diffusion-controlled evaporation of a spray is primarily influenced by its initial-droplet Sauter mean diameter and to a much lesser degree by the other pertinent mean diameters of the droplets.

In the case of radiation-controlled evaporation of sprays, Fig. 9 shows the evaporation schedules for sprays in which one of the initial-droplet mean diameters of the sprays is, in turn, kept constant. The trends of these results are the same as in the case of diffusion-controlled evaporation. However, in the case of radiation-controlled evaporation it appears that when the initial Sauter mean diameters of the droplets are the same [Fig. 9(d)], evaporation rate is practically independent of initial-droplet size distribution. Thus the radiation-controlled evaporation of sprays is solely influenced by initial-droplet Sauter mean diameter. As in the case of diffusion-controlled evaporation of sprays, a lower initial Sauter mean diameter causes a higher rate of evaporation of the spray.

Consider now the representation of a polydisperse spray by a monodisperse spray of droplet size equal to the initial Sauter mean diameter of the polydisperse spray droplets. The temporal variation of the volume of unevaporated droplets of the monodisperse spray is given by:

$$\frac{V(t)}{V(0)} = \left[\frac{\bar{x}_{32}(t)}{\bar{x}_{32}(0)}\right]^3 = [1 - \tau_{32}]^{3/(k+1)}. \qquad (36)$$

Tables 2 and 3 compare the variations of the volume fraction of unevaporated droplets with time, calculated from the above monodisperse model, with the corresponding results of two polydisperse sprays for the cases of diffusion-controlled and radiation-controlled evaporation respectively. The two polydisperse sprays considered conform to the Nukiyama–Tanasawa distribution ($\varepsilon = 2$, $\gamma = 1.0$) and the Hiroyasu–Kadota distribution ($\varepsilon = 0$, $\gamma = 1.0$). In addition, Table 2 includes results obtained using Tanasawa's empirical expression for burning of a spray (see Shipinski *et al.* [35]). The volume fraction of evaporated (unburned) droplets given by Tanasawa's empirical expression is:

$$\frac{V(t)}{V(0)} = \exp\left[-\left(\frac{3}{4}\tau_{32}\right)^{0.75}\left(1 - \frac{\tau_{32}}{4}\right)^{-0.25}\right]. \qquad (37)$$

Table 2 shows that the monodisperse model based on the initial Sauter mean diameter of the spray gives good agreement during the early stages of evaporation. This agreement degenerates, however, as evaporation proceeds. The error in the volume fraction of unevaporated droplets calculated by the monodisperse model increases with increased evaporation time. When 40% of the volume of droplets has evaporated, the error of the monodisperse model is about 20%. Bracco [36] found that a monodisperse spray model using an initial droplet size of 0.78 $\bar{x}_{30}(0)$ gave acceptable results for the combustion of an enthanol spray in a constant-area rocket motor. This spray was assumed to have a Nukiyama–Tanasawa ($\varepsilon = 2.0$, $\gamma = 1.0$) initial size distribution. Note that for this distribution, the initial Sauter mean diameter, $\bar{x}_{32}(0)$, is equal to 0.78 $\bar{x}_{30}(0)$ (see Table 1).

In the case of radiation-controlled evaporation, shown in Table 3, the monodisperse model agrees much better with the evaporation of the polydisperse sprays than in the diffusion-controlled case. Here the error in the volume fraction of unevaporated droplets calculated with the monodisperse model based on the initial Sauter mean diameter of the spray droplets does not reach 20% until about 70% of the volume of the droplets is evaporated.

CONCLUSIONS

(1) The rate of evaporation of a polydisperse spray is primarily influenced by the initial-droplet Sauter mean diameter and to a much lesser degree by the other initial-droplet mean diameters of the spray.

(2) For diffusion-controlled evaporation, compa-

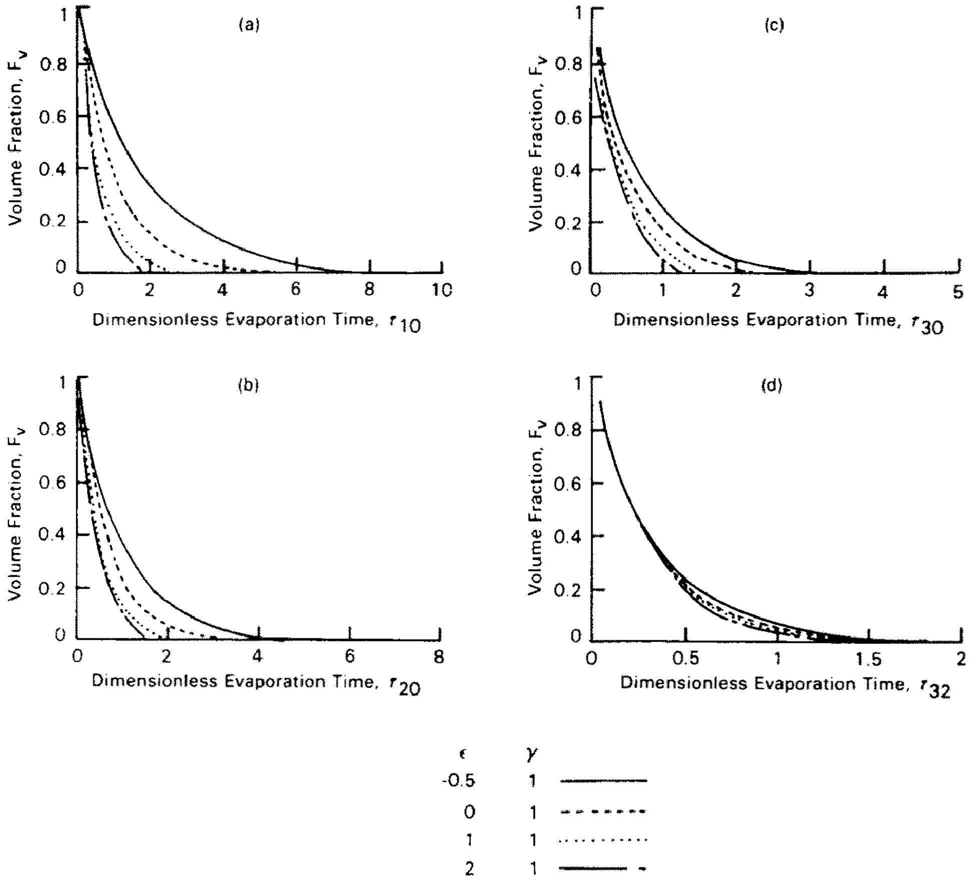

Fig. 9. Comparisons of the radiation-controlled evaporation of sprays of various initial size distributions. (a) Same initial linear mean diameters; (b) same initial surface mean diameters; (c) same initial volume mean diameters; (d) same initial Sauter mean diameters.

Table 2. Diffusion-controlled evaporation of polydisperse sprays using a monodisperse spray model

	Volume fraction of unevaporated droplets, F_v			
Evaporation time τ_{32}	Nukiyama–Tanasawa distribution $\varepsilon = 2, \quad \gamma = 1$	Hiroyasu–Kadota distribution $\varepsilon = 0, \quad \gamma = 1$	Tanasawa's model	Monodisperse model $x(0) = \bar{x}_{32}(0)$
0.05	0.938	0.931	0.918	0.926
0.10	0.879	0.871	0.857	0.854
0.20	0.770	0.769	0.783	0.715
0.30	0.675	0.685	0.717	0.586
0.40	0.593	0.615	0.660	0.465
0.50	0.522	0.554	0.609	0.354
0.60	0.461	0.502	0.564	0.253
0.70	0.407	0.456	0.524	0.164
0.80	0.361	0.416	0.486	0.089
0.90	0.320	0.380	0.452	0.032
1.00	0.285	0.348	0.421	0.000
1.20	0.226	0.295	0.364	
1.40	0.181	0.251	0.315	
1.60	0.146	0.216	0.272	
1.80	0.118	0.186	0.234	
2.00	0.096	0.162	0.200	
3.00	0.006	0.084	0.074	

Table 3. Radiation-controlled evaporation of polydisperse sprays using a monodisperse spray model

Evaporation time τ_{32}	Volume fraction of unevaporated droplets, F_v		
	Nukiyama–Tanasawa distribution $\varepsilon = 2, \quad \gamma = 1$	Hiroyasu–Kadota distribution $\varepsilon = 0, \quad \gamma = 1$	Monodisperse model $x(0) = \bar{x}_{32}(0)$
0.05	0.859	0.861	0.857
0.10	0.735	0.741	0.729
0.20	0.533	0.549	0.512
0.30	0.382	0.407	0.343
0.40	0.271	0.301	0.216
0.50	0.190	0.223	0.125
0.60	0.132	0.165	0.064
0.70	0.091	0.122	0.027
0.80	0.062	0.091	0.008
0.90	0.042	0.067	0.001
1.00	0.029	0.050	0.000
1.20	0.013	0.027	
1.40	0.006	0.015	
1.60	0.002	0.008	
1.80	0.001	0.004	
2.00	0.000	0.002	

rison of the evaporation rate of a polydisperse spray with that of a monodisperse spray with droplet size equal to the initial-droplet Sauter mean diameter of the polydisperse spray shows reasonable agreement. This agreement, which is excellent during the early stages of evaporation, deteriorates as evaporation proceeds.

(3) For radiation-controlled evaporation (opaque droplets), the representation of a polydisperse spray by a monodisperse spray of droplet size equal to the initial-droplet Sauter mean diameter of the polydisperse spray is an excellent assumption.

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INFLUENCE DES PARAMETRES DE DISTRIBUTION DE TAILLE SUR L'EVAPORATION DES BROUILLARDS POLYDISPERSES

Résumé—Une étude analytique porte sur l'évaporation controlée par la diffusion et par le rayonnement (gouttelettes opaques) de brouillards polydispersés d'un composant unique. Les caractéristiques globales de l'évaporation pour des brouillards à différentes distributions initiales de taille sont mieux représentées par le diamètre initial moyen de Sauter. Dans le cas de l'évaporation controlée par la diffusion, la représentation d'un brouillard polydispersé par un brouillard monodispersé de gouttelettes ayant un diamètre égal au diamètre moyen de Sauter du brouillard polydispersé est une bonne hypothèse pendant les premières étapes de l'évaporation. Lorsque le volume des gouttelettes est évaporé au taux de 40%, l'erreur atteint 20% pour le modèle monodispersé. D'autre part, dans le cas de l'évaporation controlée par le rayonnement, le modèle du brouillard monodispersé donne de bons résultats pour presque toute la période d'évaporation du brouillard.

DER EINFLUSS VON TROPFENGRÖSSENVERTEILUNGS-PARAMETERN AUF DIE VERDUNSTUNG POLYDISPERSER VERDÜNNTER SPRÜHNEBEL

Zusammenfassung—Es wurde eine analytische Untersuchung der durch Diffusion und auch der durch Strahlung (trübe Tröpfchen) bedingten Verdunstung von polydispersen Einkomponenten-Sprühnebeln durchgeführt. Es stellte sich heraus, daß sich das Verdunstungsverhalten von Sprühnebeln mit unterschiedlichen Anfangstropfengrößen-Verteilungen am besten mit dem anfänglichen Sauter-Durchmesser korrelieren läßt. Für den Fall der durch Diffusion bedingten Verdunstung ist die Darstellung eines polydispersen durch einen monodispersen Sprühnebel mit einer dem anfänglichen Sauter-Durchmesser des polydispersen Sprühnebels entsprechenden Tropfengröße eine für die Anfangsstadien der Verdunstung gut zutreffende Annahme.

Wenn 40% des Tropfenvolumens verdunstet sind, erreicht der Fehler des monodispersen Modells 20%. Andererseits erzielt man im Fall der durch Strahlung bedingten Verdunstung mit dem monodispersen Sprühnebel-Modell für den Hauptteil des Verdunstungsvorgangs gute Ergebnisse.

ВЛИЯНИЕ ПАРАМЕТРОВ РАСПРЕДЕЛЕНИЯ КАПЕЛЬ ПО РАЗМЕРАМ НА ИСПАРЕНИЕ ПОЛИДИСПЕРСНЫХ РАЗБАВЛЕННЫХ КАПЕЛЬНЫХ ВЗВЕСЕЙ

Аннотация — Проведено аналитическое исследование диффузионного и радиационного (капли непрозрачные) испарения однокомпонентных полидисперсных капельных взвесей. Найдено, что суммарные характеристики испарения взвесей с различными начальными распределениями капель по размерам лучше всего описываются с помощью начального среднего диаметра Саутера. При диффузионном испарении взвеси полидисперсную капельную взвесь на начальных стадиях испарения можно с хорошей степенью точности представить как монодисперсную с размером капель, равным начальному среднему диаметру Саутера для полидисперсной взвеси. После испарения 40% объёма капель, погрешность, вносимая за счёт использования монодисперсной модели, достигает 20%. С другой стороны, при испарении под воздействием излучения модель монодисперсной взвеси даёт хорошие результаты почти для всего периода испарения капельной взвеси.